Mixed model for temperature structure functions in fully developed turbulence

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A mixed model for temperature structure functions in fully developed turbulent shear flows is constructed on the assumptions that the statistics of the energy dissipation rate ϵ is log-Poisson and the temperature dissipation rate N can be described by the β model. The temperature structure functions predicted by this mixed model are in excellent agreement with the experimental data reported in the literature. [S1063-651X(98)08112-4]

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According to the Kolmogrov 1941 (K41) theory for fully developed turbulence [1], there is a universal state in which the velocity difference δu across a distance l has a scaling behavior when l is in the inertial range $\langle|\delta u(l)|^p\rangle \sim l^{\zeta_p}$ with $\zeta_p = p/3$. In the K41 theory the energy dissipation rate ϵ was assumed constant. However, experimental results confirm the existence of the intermittency of ϵ and ζ_p substantially deviates from the value predicted by the K41 theory at large values of p.

Many models have been brought out for the correction of the K41 theory by taking the intermittency into consideration [2]. It is now believed that in the inertial range the structure functions of the energy dissipation rate ϵ also follow a scaling law, i.e., $\langle|\epsilon(l)|^p\rangle \sim l^{\tau_p}$. According to Kolmogrov's refined similarity hypothesis, $\epsilon \sim \delta u^3/l$ and then $\delta u \sim (l\epsilon)^{1/3}$, $(\delta u)^p \sim \epsilon^{p/3} l^{p/3}$, and $\zeta_p = p/3 + \tau_{p/3}$. The validity of the intermittency models can be checked through a comparison of the scaling exponents with experimental results.

A quantized energy cascade model recently proposed by She and Leveque leads to a prediction of ζ_p in excellent agreement with experiment results [3]. She and Waymire [4] and Dubrulle [5] independently observed that the model corresponds to a log-Poisson distribution. In this model, the cascade multiplicative factor $W_{l_1 l_2}$ for any arbitrary pair of length scale l_1, l_2 can be expressed as [5]

$$W_{l_1 l_2} = (l_1/l_2)^{2/3} (\tfrac{2}{3})^X, \tag{1}$$

where X obeys Poisson law

$$P(X=m) = \exp[-\ln(l_1/l_2)^2] \ln^m (l_1/l_2)^2/m! \tag{2}$$

The scaling exponents of the structure functions of the energy dissipation rate and the velocity predicted by this model are [3,5]

$$\tau_p = -2p/3 + 2[1-(\tfrac{2}{3})^p], \tag{3}$$

$$\zeta_p = p/9 + 2[1-(\tfrac{2}{3})^{p/3}]. \tag{4}$$

Comparisons of this model with experimental results were given by She and Waymire [4] and Frisch [2]. This model not only agrees excellently with experimental results, but also bases itself on firm physical arguments and has no adjustable parameters like in other models, which either have no physical meaning or cannot be determined by plausible physical arguments.

Like velocity in the K41 theory, the temperature in a fully developed turbulence was also analyzed in an analogous manner by Obuhov [1]. Similarly, the structure functions of the temperature difference $\delta\theta$ across a distance l are assumed to have a scaling behavior when l is in the inertial range $\langle|\delta\theta(l)|^p\rangle \sim l^{\alpha_p}$. According to Kolmogrov's refined similarity hypothesis $N \sim \delta u\, \delta\theta^2/l$, we have

$$(\delta\theta)^p \sim N^{p/2} \epsilon^{-p/6} l^{p/3}. \tag{5}$$

If ϵ and N are constants we have $\alpha_p = p/3$. As expected, experimental results show that α_p greatly deviates from $p/3$.

There are also some models that have been introduced to solve the problem, for example, the β model [6], the joint log-normal model for ϵ and N [7], and the cascade model with the temperature variance following an exponential distribution [8]. Unlike the situation for the velocity, however,

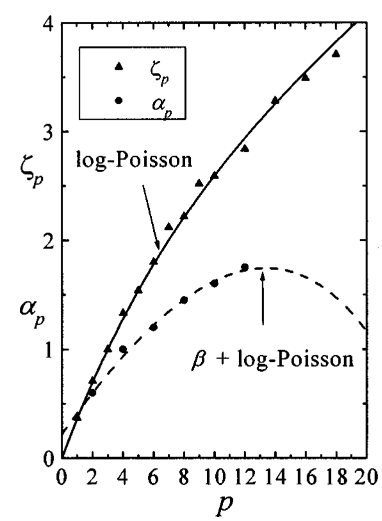

FIG. 1. Comparison of scaling exponents for temperature and velocity structure functions. Experiments: ●, α_p [8], ▲, ζ_p [9]. Models: ——, ζ_p (SL model); − − −, α_p (mixed model proposed in this paper).

none of these models predicts scaling exponents agreeing well with experimental results, especially for large values of p.

Experiments show that the dissipation field of passive scalars in turbulence is also intermittent [9]. Taking the intermittency of both ϵ and N into consideration and assuming that N also follows a scaling law in the inertial range $\langle|N(l)|^p\rangle \sim l^{\eta_p}$, we have immediately from Eq. (5) that the scaling exponents α_p of temperature structure functions depend on the scaling properties of both ϵ and N,

$$\alpha_p = p/3 + \eta_{p/2} + \tau_{-p/6}. \tag{6}$$

It is straightforward to adopt the log-Poisson model for the ϵ exponent $\tau_{-p/6}$. The problem arises for the N exponent $\eta_{p/2}$. If we assume the same scaling behavior of N as ϵ, then we have $\alpha_p = \zeta_p$. Experimental results show that there is a great discrepancy between these two exponents. This means that the scaling behavior of N is obviously different from that of ϵ. As ϵ and N relate to different physical processes, it is difficult to make similar arguments. Here we do not carry out work on the difference between ϵ and N.

Let us just recall that the β model has a clear physical cascade picture and a rather long history [2]. It is worth trying this model for the description of N. According to this model, the cascade multiplicative factor $W_{l_1 l_2}$ for successive pair of length scale $l_1 = 2l_2$ can be expressed as

$$W_{l_1 l_2} = \begin{cases} 0 & \text{with probability } 1-\beta \\ 1/\beta & \text{with probability } \beta \end{cases} \tag{7}$$

where $\beta = 2^{-(3-D)} = 2^{-\mu}$ and D is the self-similarity dimension. The scaling exponents η_p of the structure functions of the temperature dissipation rate N predicted by this model are [6]

$$\eta_p = \mu(1-p), \tag{8}$$

where μ is widely used to characterize the intermittency correction to the K41 theory. Experimental measurements give a value between 0.2 and 0.25. We choose $\mu = 2/9$ according to the results of She and Leveque [3]. Thus we have

$$\alpha_p = p/3 + \tfrac{2}{9} - 2[(\tfrac{3}{2})^{p/6} - 1]. \tag{9}$$

A comparison of the predictions of α_p by Eq. (9) (dashed line in Fig. 1 with the label β+log-Poisson) with the experimental results of Antonia *et al.* [7] up to an order of 12 for $R_\lambda = 850$ is given in Fig. 1. It can be seen clearly that the model predictions are in excellent agreement with experimental results. It should be noted that direct numerical simulation results are in good agreement with experimental data [10]. In the figure a comparison is also given for the velocity exponent ζ_p predicted by Eq. (2) with the experimental results of Anselmet *et al.* [11] up to an order of 18 for $R_\lambda = 852$.

We note that although in the range of experimental data the agreement is excellent, just beyond this range the exponent α_p begins to decrease with increasing order p. While there is no experimental evidence of this decreasing tendency for both ζ_p and α_p, all other models also show a rapid decrease of α_p with p at not very large values of p. Conformity of this tendency requires further experimental measurements.

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